

## Appendix A

### Sample Instructional Profile

#### Instructional Profile: Adding and Subtracting Fractions with Unlike Denominators, Grades Four Through Seven

Adding and subtracting fractions with unlike denominators has been selected for elaboration because:

- The student is required to use a number of discrete skills to solve these problems.
- There are specific types of problems within this category, each with its own particular prerequisite, and some problems are much more complex than others.

These types of problems and the prerequisites for solving them need to be introduced and practiced for varying amounts of time. The teacher has to be adequately prepared to present each concept for solving problems when the students are ready for it.

#### Standards for Adding and Subtracting Fractions

The standards in this section present the content and the related concepts and skills that students need to be able to add and subtract fractions with unlike denominators.

*Grade 5. Number Sense 2.3.* Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

*Grade 6. Number Sense 2.1.* Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

*Related standards.* To solve the problems required by the content standards listed previously, students need to learn related concepts and skills:

- Compute and perform simple multiplication of fractions.
- Rewrite a fraction as an equivalent fraction with a different denominator.
- Reduce fractions.
- Convert improper fractions to mixed numbers.
- Determine the operation called for in a story problem or in a concrete situation in which fractions appear and create a problem to be solved.

## Considerations for Instructional Design

The following considerations for instructional design are discussed in this section: sequence of instruction, teaching the components of complex applications, selection of examples, and introduction of concepts and skills.

### Sequence of Instruction

This section presents procedures and cautions for teachers to follow when they introduce addition and subtraction of fractions with unlike denominators.

1. As mentioned in the discussion for grades four and five in Chapter 3, the teacher should rely on

$$\frac{a}{b} \pm \frac{c}{d} = \frac{(ad \pm bc)}{bd}$$

as the basic formula for adding or subtracting fractions. However, there are occasions when looking for a smaller common denominator of two given fractions can simplify the computation. In such cases the concept of the greatest common factor and the lowest common multiple would be discussed. Many students find these concepts confusing because both deal with factors and multiples of numbers. Care should therefore be exercised in differentiating between the two. One way is not to teach these two concepts in succession.

2. Easier problems are to be taught before more difficult ones. While this guideline seems self-evident, its application requires identifying subtypes of a general set of problems. For example, problems that require borrowing ( $2\frac{1}{2} - 1\frac{3}{4}$ ) are slightly more difficult than problems that do not require borrowing ( $2\frac{3}{4} - 1\frac{1}{2}$ ).

There are a number of distinct types of problems for adding and subtracting fractions with unlike denominators:

- Simple problems in which one fraction has the common denominator:

$$\frac{1}{2} + \frac{3}{4}$$

- Simple problems in which the common denominator is neither denominator:

$$\frac{3}{4} - \frac{1}{5}$$

- Problems with mixed numbers—no regrouping is required:  $5\frac{1}{5} + 3\frac{2}{3}$
- Problems that require adding three fractions:

$$\frac{1}{3} + \frac{1}{5} + \frac{2}{3}$$

- Problems that require subtracting a mixed number from a whole number ( $8 - 3\frac{1}{4}$ ) and adding mixed numbers: ( $7\frac{3}{4} + 2\frac{4}{5}$ )
- Mixed-number problems that require regrouping ( $5\frac{1}{2} - 2\frac{3}{4}$ ) and rewriting the numerator and denominator

3. Introducing too much information in a single lesson or over a short period of time can result in student confusion. The introduction of new skills and

applications needs to be controlled so that children at a particular skill level can reasonably absorb the new information.

### Teaching the Components of Complex Applications

Sometimes simple developmental approaches can help reinforce students' ability to learn a complex skill. These approaches may not be appropriate for introducing the skill itself because they are logically out of order (see the examples that follow), but they nevertheless cast a different light on the skill and therefore increase students' understanding.

#### *Example: Fractions That Equal One*

The concept of equivalent fractions has to be taught before the multiplication of fractions is introduced. But *once students have learned to multiply fractions*, they can, with hindsight and from a different perspective, look at equivalent fractions and other more elementary skills concerning fractions:

- **Equivalent fractions.** One can see why, for example,  $\frac{3}{4}$  must be equal to  $\frac{6}{8}$  by considering  $\frac{3}{4} \times 1$ :

$$\frac{3}{4} = \frac{3}{4} \times 1 = \frac{3}{4} \times \frac{2}{2} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

Similar statements can be made for any fraction  $\frac{a}{b}$ .

- **Converting improper fractions to mixed numbers.** The fraction  $\frac{13}{5}$  can be rewritten as  $2\frac{3}{5}$  because on the number line  $\frac{13}{5}$  is positioned at a point that is  $\frac{3}{5}$  beyond 2. But again, using the fact that  $1 = \frac{5}{5}$ , we can also look at this conversion differently:

$$\frac{13}{5} = \frac{5 + 5 + 3}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = 1 + 1 + \frac{3}{5} = 2\frac{3}{5},$$

where the last equality is by convention; that is, the writing of the plus sign “+” is omitted in  $2 + \frac{3}{5}$ .

- **Reducing fractions.** Again one makes use of the fact that  $1 = \frac{n}{n}$  for any non-zero integer  $n$  and that every integer has a prime decomposition to reduce a fraction to lowest terms:

$$\frac{12}{18} = \frac{2 \times 2 \times 3}{2 \times 3 \times 3} = \frac{2}{3} \times \frac{2}{2} \times \frac{3}{3} = \frac{2}{3} \times 1 \times 1 = \frac{2}{3}.$$

### Selection of Examples

Examples in teaching sets should be designed and selected to rule out possible misinterpretations. For example, if a teacher teaching students to solve application problems containing the words “younger than” presents only subtraction problems as examples, the students might develop the idea that the phrase “younger than” always involves subtraction. (*Jan is 19 years old. Alice is 7 years younger than Jan. How old is Alice?*) The students would likely miss a problem that calls for addition, such as *Marcus is 25 years old. He is 7 years younger than his brother. How old is Marcus's brother?*

Teachers should not assume that all students will automatically be able to generalize to new types of problems. For example, when students start working on new types of verbal problems requiring fractions, the teacher should point out the following facts: (1) the problems differ from problems they are familiar with; (2) the problems involve fractions; and (3) the students must be careful when setting up the problems.

*Example: Concrete Applications*

Number Sense Standard 2.3 for grade five states that students are to “solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.”

Once children have learned to compute answers to problems involving the addition and subtraction of fractions and have had sufficient practice to work the problems without prompting, applications such as the following can be introduced:

- *The recipe calls for  $\frac{3}{8}$  of a pound of nuts. Anna has only  $\frac{1}{4}$  of a pound of nuts in her kitchen. If she goes to the store to buy nuts, what fraction of a pound of nuts will she need?*
- *There is  $\frac{2}{3}$  of a pizza left over, and  $\frac{1}{4}$  of another equal-sized pizza is left over. If we put the pieces from both pizzas together, what part of a whole pizza would we have?*

**Introduction of Concepts and Skills**

Initial teaching should be interactive. The teacher not only demonstrates and explains but also asks frequent questions to check for understanding.

*Example: Initial Strategy for Adding and Subtracting Fractions with Unlike Denominators*

It must be emphasized that the basic definition of the adding or subtracting of fractions is simple and direct. It is only the refinements of the definition that cause complications. Students should be told that the *definition is more basic than the refinements*.

The basic *definition* is to multiply the denominators of the fractions to make a common denominator. For example, the addition problem of  $\frac{3}{4} + \frac{2}{5}$  is done in the following way to obtain the answer  $\frac{23}{20}$ :

$$\begin{aligned} & \frac{3}{4} + \frac{2}{5} \\ &= \frac{3 \times 5}{4 \times 5} + \frac{2 \times 4}{5 \times 4} \\ &= \frac{15}{20} + \frac{8}{20} \\ &= \frac{23}{20} \end{aligned}$$

Sometimes, it is possible by inspection to decide on a common denominator of two fractions that is smaller than the product of the two denominators.

Thus  $\frac{5}{6} + \frac{1}{8}$  equals  $\frac{23}{24}$  because visibly 24 is a common multiple of the two given denominators 6 and 8. One proceeds to rewrite the two fractions with 24 as the denominator:

$$\frac{5 \times 4}{6 \times 4} + \frac{1 \times 3}{8 \times 3} = \frac{20}{24} + \frac{3}{24} = \frac{23}{24}$$

However, students should also be told that if they add  $\frac{5}{6} + \frac{1}{8}$  using the basic definition (see p. 276) to obtain

$$\frac{5}{6} + \frac{1}{8} = \frac{40}{48} + \frac{6}{48} = \frac{46}{48},$$

then the resulting answer  $\frac{46}{48}$  is as valid as  $\frac{23}{24}$ . (Note that  $\frac{46}{48}$  would be incorrect only if a reduced fraction is specifically requested for the answer.)

Similarly, the use of the basic definition in the subtraction of two fractions

$$\frac{13}{42} - \frac{2}{35} = \frac{13 \times 35}{42 \times 35} - \frac{2 \times 42}{35 \times 42} = \frac{455}{1470} - \frac{84}{1470} = \frac{371}{1470}$$

yields an answer that is just as valid as factoring the denominators and finding the least common multiple:

$$\begin{aligned} \frac{13}{42} - \frac{2}{35} &= \frac{13}{2 \times 3 \times 7} - \frac{2}{5 \times 7} \\ &= \frac{13 \times 5}{2 \times 3 \times 7 \times 5} - \frac{2 \times 3 \times 2}{2 \times 3 \times 7 \times 5} \\ &= \frac{65}{210} - \frac{12}{210} = \frac{53}{210} \end{aligned}$$

The advantage of the least common multiple procedure therefore lies in avoiding the computations with relatively large numbers.

In context, any one of these strategies may be selected for use if it happens to be the most convenient. Students should always be taught the basic definition (see p. 276) first, but the order of teaching the other strategies can vary. Ultimately, they must know all three approaches.